WFM01/01: Further Pure Mathematics F1

Question Number	Scheme		Marks
1. (a)	$\dfrac{z_1}{z_2} = \dfrac{2+8i}{1-i} \times \dfrac{1+i}{1+i}$		M1
	$= \dfrac{2+2i+8i-8}{2} = -3+5i$		A1 A1
			(3)
(b)	$\left\lvert\dfrac{z_1}{z_2}\right\rvert = \sqrt{(-3)^2+5^2} = \sqrt{34}$	(or awrt 5.83)	M1 A1ft
			(2)
(c)	$\tan\alpha = -\dfrac{5}{3}$ or $\dfrac{5}{3}$		M1
	$\arg\dfrac{z_1}{z_2} = \pi - 1.03... = 2.11$		A1
			(2) **(7 marks)**
2. (a)	$f(1.6) = -1.29543081...$	awrt -1.30	B1
	$f(1.8) = 0.5401863372$	awrt 0.54	B1
	$\dfrac{\alpha - 1.6}{\text{"}1.29543081...\text{"}} = \dfrac{1.8-\alpha}{\text{"}0.5401863372\text{"}}$		M1
	$\alpha = 1.741143899$	awrt 1.741	A1
			(4)
(b)	$f'(x) = 10x - 6x^{\frac{1}{2}}$		M1 A1
	$f(1.7) = -0.4161152711...$	awrt -0.42	B1
	$f'(1.7) = 9.176957114...$	awrt 9.18	B1
	$\alpha_2 = 1.7 - \dfrac{f(1.7)}{f'(1.7)}$		M1
	$\alpha_2 = 1.745$	cao	A1
			(6) **(10 marks)**

WFM01/01: Further Pure Mathematics F1

Question Number	Scheme	Marks
3. (a)	$PQ = 12 \Rightarrow$ By symmetry $y_p = \frac{12}{2} = 6$	B1
		(1)
(b)	$y^2 = 8x \Rightarrow 6^2 = 8x$	M1
	$\Rightarrow x = \frac{36}{8} = \frac{9}{2}$	A1
		(2)
(c)	Focus $S(2, 0)$	B1
	Gradient $PS = \frac{6-0}{\frac{9}{2}-2}\left(=\frac{6-0}{\frac{9}{2}-2}=\frac{12}{5}\right)$	M1
	Either $y-0=\frac{12}{5}(x-2)$ or $y-6=\frac{12}{5}(x-\frac{9}{2})$	M1
	Or $y=\frac{12}{5}x+c$ and $0=\frac{12}{5}(2)+c \Rightarrow c=-\frac{24}{5}$	
	l: $\underline{12x-5y-24=0}$	A1
		(4) **(7 marks)**
4. (a)	$\alpha+\beta=\frac{4}{5},\quad \alpha\beta=\frac{1}{5}$	B1, B1
		(2)
(b)	$\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=\frac{\alpha^2+\beta^2}{\alpha\beta}$	B1
	$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$	B1
	$\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=\frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}=\frac{\frac{16}{25}-\frac{2}{5}}{\frac{1}{5}}$	M1
	$=\frac{6}{5}$ *	A1
		(4)
(c)	$\alpha+\frac{1}{\alpha}+\beta+\frac{1}{\beta}=\alpha+\beta+\frac{\alpha+\beta}{\alpha\beta}=\frac{24}{5}$	M1 A1
	$\left(\alpha+\frac{1}{\alpha}\right)\left(\beta+\frac{1}{\beta}\right)=\alpha\beta+\frac{\alpha}{\beta}+\frac{\beta}{\alpha}+\frac{1}{\alpha\beta}=\frac{1}{5}+\frac{6}{5}+5=\frac{32}{5}$	M1 A1
	$x^2-\frac{24}{5}x+\frac{32}{5}=0 \Rightarrow 5x^2-24x+32=0$	M1 A1
		(6) **(12 marks)**

WFM01/01: Further Pure Mathematics F1

Question Number	Scheme	Marks
5.	$f(1) = 5 + 8 + 3 = 16$, (which is divisible by 4). ($\therefore$ True for $n = 1$).	B1
	Assume true for f(k)	
	Using the formula to write down f(k + 1), $f(k+1) = 5^{k+1} + 8(k+1) + 3$	B1
	$f(k+1) - f(k) = 5^{k+1} + 8(k+1) + 3 - 5^k - 8k - 3$	M1
	$= 5(5^k) + 8k + 8 + 3 - 5^k - 8k - 3 = 4(5^k) + 8$	A1
	$f(k+1) = 4(5^k + 2) + f(k)$, **which is divisible by** 4	A1
	$\therefore$ True for $n = k + 1$ **if** true for $n = k$. True for $n = 1$, $\therefore$ true for all n.	A1 **(6 marks)**
6. (a)	$r(r+1)(r+3) = r^3 + 4r^2 + 3r$, so **use** $\sum r^3 + 4\sum r^2 + 3\sum r$	M1
	$= \frac{1}{4}n^2(n+1)^2 + 4\left(\frac{1}{6}n(n+1)(2n+1)\right), + 3\left(\frac{1}{2}n(n+1)\right)$	A1, A1
	$= \frac{1}{12}n(n+1)\{3n(n+1) + 8(2n+1) + 18\}$ or $= \frac{1}{12}n\{3n^3 + 22n^2 + 45n + 26\}$	M1
	or $= \frac{1}{12}(n+1)\{3n^3 + 19n^2 + 26n\}$	A1
	$= \frac{1}{12}n(n+1)\{3n^2 + 19n + 26\} = \frac{1}{12}n(n+1)(n+2)(3n+13)$ ($k = 13$)	M1 A1
		(7)
(b)	$\sum_{21}^{40} = \sum_{1}^{40} - \sum_{1}^{20}$	M1
	$= \frac{1}{12}(40 \times 41 \times 42 \times 133) - \frac{1}{12}(20 \times 21 \times 22 \times 73), = 707210$	A1, A1
		(3) **(10 marks)**

WFM01/01: Further Pure Mathematics F1

Question Number	Scheme	Marks
7. (a)	$y = \frac{36}{x} \Rightarrow \frac{dy}{dx} = -36x^{-2}$	M1
	At $\left(6t, \frac{6}{t}\right)$, $\frac{dy}{dx} = -\frac{c^2}{(6t)^2} = -\frac{1}{t^2}$	M1 A1
	$y - \frac{6}{t} = -\frac{1}{t^2}(x - 6t) \quad \Rightarrow y = -\frac{1}{t^2}x + \frac{12}{t}$ (*)	M1 A1cso
		(5)
(b)	Substitute $(-9, 12)$: $12 = -\frac{1}{t^2}(-9) + \frac{12}{t}$	M1
	$12t^2 - 12t - 9 = 0$	A1
	$(2t-3)(2t+1) = 0 \quad \Rightarrow \quad t = \frac{3}{2} \quad t = -\frac{1}{2}$	M1 A1
	$t = \frac{3}{2} \quad t = -\frac{1}{2} \Rightarrow$ Points are $(9, 4)$ and $(-3, -12)$	M1 A1 A1
		(7) **(12 marks)**
8. (i)		
(a)	$120°$ or $\frac{2\pi}{3}$ rotation about the origin, anticlockwise.	B1, B1
		(2)
(b)	$\begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix}$	B1
		(1)
(c)	$\mathbf{R} = \begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix} = \begin{pmatrix} -\frac{3}{2} & -\frac{3\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix}$	M1 A1 A1 (-1 each error)
		(3)
(ii) (a)	$\det \mathbf{S} = 1\times1 - 3\times -3 (= 10)$ *or* $3^2 + 1^2 (= 10) \Rightarrow$ Enlargement scale factor $= \sqrt{10}$	M1 A1
		(2)
(b)	$\tan\theta = \frac{3}{1} \Rightarrow \theta = 71.6^o$, anticlockwise.	M1 A1, A1
		(3) **(11 marks)**